Imaging in turbid media using modulation frequency scanning

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We propose an imaging scheme to detect an object inside a turbid medium using intensity modulated waves as a probe. We show how the new degree of freedom represented by the modulation frequency allows sufficient control to reconstruct the image from the scattered wave signal.

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Using electromagnetic and other waves to probe the interior of turbid materials, especially human or other biological tissue, has been an important application of physics since Roentgen's discovery of x rays more than a century ago. Early work in transmission radiography has advanced to sophisticated computed tomography, while parallel advancements in magnetic resonance and ultrasonic imaging have become the staples of modern diagnostic imaging in medicine [1] and nondestructive sample evaluation in industry [2]. Such techniques can be classified into four broad categories depending on the underlying physical principle on which image reconstruction is based as well as the nature of the probing signal. Transmission methods, such as x rays, utilize the integrated attenuation of the imaging beam along lines of sight, either to produce a two-dimensional map of the integrated absorption properties of the medium, or to produce a three-dimensional map by utilizing many lines of sight and back-projection algorithms. The latter method can also use interior sources, as is the case with nuclear imaging such as positron emission tomography. Wave echo methods, such as ultrasonic imaging, send short wave trains into the medium and use the delay time of singly reflected echoes to reconstruct the spatially varying density of the medium. The wavelength inside the medium largely determines the resolution of the image. Resonance methods, such as magnetic resonance imaging (MRI), excite responses at many locations within the medium and use the temporal structure of the resultant signal to determine both the density of resonant sites and information concerning the local (in the case of MRI, magnetic) environment of the site. Optical fluorescence methods are relatively more simple implementations of this scheme. Note that in these three methods, the wave properties of the source radiation are either ignored or utilized only marginally.

In wave scattering techniques, information is gleaned from the complicated scattered signal, utilizing both amplitude and phase information as input to an often nontrivial inversion algorithm. Well-developed examples of this general technique include quantum scattering, in which the scattered wave signal is used to infer the quantum state of the target, and holography, in which information collected from the scattered signal is used in a subsequent experiment to recreate an image of the original target. It is clear that wave scattering techniques are in general quite powerful, but also difficult, owing to the necessity for often nontrivial measurement of both amplitude and phase information and the extraction of the desired information via equally nontrivial physical or analytical/computational manipulation of the measured data. Indeed, general analytical or computational solutions of the inverse problem [3] for waves scattered from turbid media are at the forefront of current research.

In this paper we propose a method for using wave scattering signals to extract information from turbid media that also includes elements related to the wave echo technique. It exploits the degree of freedom contained in the modulation frequency in a similar way that current methods use multiple detector locations to exploit the spatial degree of freedom. We demonstrate that if the scattered data are obtained for a range of modulation frequencies these data can be used directly to reconstruct an image. We propose both an analytical inversion scheme, with promise for wider application, together with computational results for a simple example system. A number of interesting challenges can now be addressed, and we discuss these at the end.

The propagation of energy through a turbid medium can be mathematically approximated in terms of the probability density $I(r,\Omega,t)$ by the usual Boltzmann energy transport equation [4]

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\Omega\cdot\nabla\right)I(r,\Omega,t)=-(\mu_s+\mu_a)I(r,\Omega,t)+\mu_s\int d\Omega' p(\Omega,\Omega')I(r,\Omega',t). \tag{1}$$

Here $1/\mu_s$ and $1/\mu_a$ denote the length scales associated with scattering and absorption, respectively, c is the propagation speed, and $p(\Omega,\Omega')$ is the conditional probability distribution that a signal associated with the Ω' direction is scattered into the Ω direction.

To illustrate the frequency based imaging scheme as a proof of concept, we choose the simplest possible scenario for which, even in principle, standard imaging techniques based on spatially separated detectors cannot be used to reconstruct heterogeneous scattering or absorption properties of the system. This system is characterized by a turbid medium in which only forward and backward scattering is possible. For several direct realizations of this system, see [5–7]. The modulated probe signal is injected from the left side and the scattered signal is analyzed. The set of coupled differential equations for the signal fluence (intensity) along the positive and negative x directions, $I(x,\pm,t)$, can be derived from Eq. (1) using a bidirectional phase function:

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)I(x,+,t)=-[\mu(x)+\mu_a]I(x,+,t)+\mu(x)I(x,-,t), \tag{2a}$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t}-\frac{\partial}{\partial x}\right)I(x,-,t)=+\mu(x)I(x,+,t)-[\mu(x)+\mu_a]I(x,-,t). \tag{2b}$$

The position-dependent scattering coefficient is given by $\mu(x)=\overline{\mu}+\delta\mu(x)$, where $\delta\mu(x)$ represents the scattering inhomogeneity (an object embedded in the medium, for example). The absorption coefficient μ_a is assumed to be uniform in the medium of width W. The boundary conditions for this system in the frequency domain are $I(x=0,+,\omega)=f(\omega)$ and $I(x=W,-,\omega)=0$ associated with a wave source $f(\omega)$ that injects the intensity modulated signal at the left edge $(x=0)$ of the medium.

Despite the quasi-one-dimensional character of Eq. (2), even the forward problem of constructing $I(x=0,-,\omega)$ for a given $\mu(x)$ cannot be solved analytically beyond perturbation theory. In order to generate the scattered data we have used a computer simulation that mimics the experimentally measured data. We define the normalized reflection amplitude $r(\omega)\equiv I(x=0,-,\omega)/f(\omega)$. The propagator for the forward problem in the frequency domain $(\partial/\partial t=-i\omega)$ can be formally expressed as the limit of an infinite number of spatially ordered product operators

$$\begin{pmatrix} I(W,+,\omega) \\ 0 \end{pmatrix}=\lim_{\substack{\Delta x\to 0\\ N\to\infty}}\prod_{j=0}^{N}\exp[\Delta x M(x_j)]\begin{pmatrix} f(\omega) \\ r(\omega)f(\omega)\end{pmatrix}, \tag{3}$$

where each 2×2 matrix $M(x)$ is related to the position dependent generator in Fourier space, similar to that on the right-hand side of Eq. (2), and $N\Delta x=W$. The four matrix elements of the exponentiated form are given by

$$[e^{\Delta x M(x_j)}]_{1,1}=\cosh[\kappa(x_j)\Delta x]+\{i\omega/c-[\mu(x_j)+\mu_a]\}\sinh[\kappa(x_j)\Delta x]/\kappa(x_j), \tag{4a}$$

$$[e^{\Delta x M(x_j)}]_{1,2}=-[e^{\Delta x M(x_j)}]_{2,1}=\mu(x_j)\sinh[\kappa(x_j)\Delta x]/\kappa(x_j), \tag{4b}$$

$$[e^{\Delta x M(x_j)}]_{2,2}=\cosh[\kappa(x_j)\Delta x]-\{i\omega/c-[\mu(x_j)+\mu_a]\}\sinh[\kappa(x_j)\Delta x]/\kappa(x_j), \tag{4c}$$

where $\kappa(x)\equiv\sqrt{\{[i\omega/c-\mu(x_j)-\mu_a]^2-\mu(x_j)^2\}}$. Using this transfer matrix approach, we can solve the forward problem to obtain numerically the reflection amplitude $r(\omega)$ with arbitrary precision for any $\mu(x)$.

In order to derive an imaging algorithm, the forward problem must be solved in a closed form that permits inversion. This can be achieved if we assume, as in other imaging approaches, that the embedded object induces only single-scattering events, requiring that $\delta\mu$ is sufficiently small. The background $\overline{\mu}$, however, can provide a diffusive environment. We can solve the Boltzmann equation perturbatively in $\delta\mu(x)$ and obtain, after a lengthy calculation, the normalized reflection amplitude $r(\omega)$:

$$r(\omega)-\overline{r(\omega)}=\int dx\mathcal{K}(x,\omega)\,\delta\mu(x), \tag{5}$$

where the integration kernel $\mathcal{K}(x,\omega)$ can be expressed as

$$\mathcal{K}(x,\omega)\equiv\{(i\omega/c-\mu_a)xS(x)[1+\overline{r(\omega)}]+C(x)[1-\overline{r(\omega)}]\}^2, \tag{6a}$$

$$\overline{r(\omega)}\equiv-\overline{\mu}WS(W)/[-\overline{\mu}WS(W)-C(W)+W(i\omega/c-\mu_a)S(W)], \tag{6b}$$

$$S(x)\equiv\sinh\{\sqrt{[(i\omega/c-\mu_a)(i\omega/c-\mu_a-2\overline{\mu})]}x\}/\{\sqrt{[(i\omega/c-\mu_a)(i\omega/c-\mu_a-2\overline{\mu})]}x\}, \tag{6c}$$

$$C(x)\equiv\cosh\{\sqrt{[(i\omega/c-\mu_a)(i\omega/c-\mu_a-2\overline{\mu})]}x\}, \tag{6d}$$

$\overline{r(\omega)}$ can be interpreted as the reflection amplitude for a medium in the absence of any object $[\delta\mu(x)=0]$. Equation (5) converts the spatial information about the location of the object into the modulation-frequency-dependent reflection amplitude $r(\omega)$. In other words, this relationship maps uniquely the spatial inhomogeneity directly into the frequency domain. Note that in previous methods the forward problem is based on the approximate diffusion theory and the usual corresponding kernel $\mathcal{K}(x,x')$ maps the object's scattering strength at location x into the signal detected at position x' [8].

The current integration kernel $\mathcal{K}(x,\omega)$ has several interesting properties. Depending on the value of $\overline{\mu}$ it represents an entire class of different transformation schemes, which to the best of our knowledge have not been studied in the mathematical literature. In the zero-frequency limit, however, the kernel $\mathcal{K}(x,\omega=0)$ leads to a reflection coefficient that can be interpreted. For $\overline{\mu}=0$ it reduces to $\mathcal{K}(x,\omega=0)=\exp(-2\mu_a x)$ such that the dc-amplitude $r(\omega=0)$ is a direct measure for the absorption weighted integrated scattering strength of the object, $r(\omega=0)=\int dx\exp(-2\mu_a x)\delta\mu(x)$. In the opposite limit of vanishing absorption $(\mu_a=0)$, we obtain $\mathcal{K}(x,\omega=0)=\overline{\mu}W/[\overline{\mu}W+1]$, corresponding to $r(\omega=0)=\overline{\mu}W/[\overline{\mu}W+1]+\overline{\mu}W/[\overline{\mu}W+1]\int dx\ \delta\mu(x)$, proportional to the object's total scattering strength.

The echo-like character of this imaging scheme can be most clearly demonstrated for the special case of vanishing background scattering $(\overline{\mu}=0)$ where the kernel reduces to the form $\mathcal{K}(x,\omega)=\exp(-2\mu_a x+2i\omega x/c)$. The inversion can even be performed fully analytically, leading to the predicted image $\delta\mu_I(x)$ in terms of the scattered data $r(\omega)$:

$$\delta\mu_I(x)=\frac{1}{c\pi}e^{2\mu_a x}\int d\omega\exp(-2i\omega x/c)r(\omega). \tag{7}$$

This expression has a convenient interpretation with respect to the temporal form of the reflected pulse associated with a very sharp input pulse, $I(x=0,+,t)=\delta(t)$. The re-

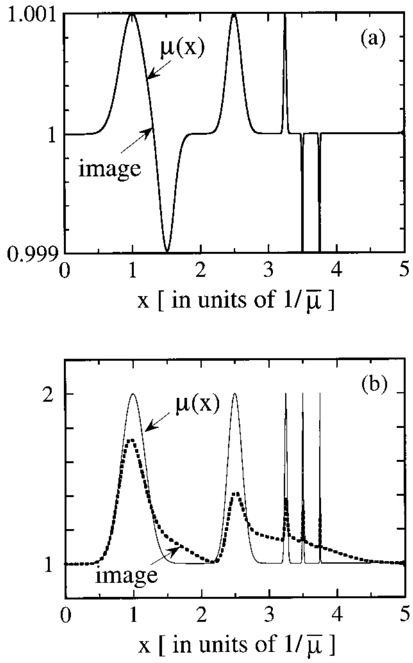

FIG. 1. The scattering coefficient $\mu(x)$ (in units of $\overline{\mu}$) as a function of the position. The dashed line represents the image obtained from the modulation-frequency-dependent scattering data using the inversion algorithm. For (a) small contrast: $\delta\mu/\overline{\mu}=0.1\%$ and (b) larger contrast: $\delta\mu/\overline{\mu}=100\%$. ($\mu_a/\overline{\mu}=0.6$, total length of the medium $W=5/\overline{\mu}$.)

flected signal profile $r(\omega)$ and $\delta\mu_I(x)$ are related by a Fourier-like transformation. In other words, the temporal pulse shape of the reflected signal at the entrance of the sample is $I(x=0,-,t)=(c/2)\,\delta\mu(x=ct/2)\exp[-\mu_a ct]$. $I(x=0,-,t)$ is a direct image of the spatial profile $\delta\mu(x)$ including the reduction due to absorption. Note that $ct/2$ is half of the round trip distance ct traveled by the wave in the medium. It reverses its direction at location $x=ct/2$ with a reflection probability directly proportional to $\delta\mu(x)$. The absorption occurs first along the distance $ct/2$ for the incoming wave and then along its return path $ct/2$ for the reflected wave. The larger the rebound probability at location x, the larger the reflected signal amplitude at the corresponding delayed time for a singly reflected echo.

In order to reconstruct the scattering inhomogeneity $\delta\mu(x)$ in the general case ($\mu_a\neq 0,\overline{\mu}\neq 0$) we have to invert the integral Eq. (5) numerically. As each eigenvalue of the integral operator is nonzero, the kernel is invertible, which is crucial for the reconstruction of an object embedded in a scattering background. This can be accomplished by discretizing the integration kernel $\mathcal{K}(x,\omega)$ on a carefully chosen space-frequency grid to generate a finite dimensional square matrix, which can be inverted numerically using standard spectral methods.

In Fig. 1(a) we show the total position-dependent scattering coefficient $\mu(x)$ of the medium as a function of the position. The dashed line corresponds to the predicted image $\mu_I(x)$ based on the reflected frequency spectrum. The two profiles are indistinguishable. Even the two narrowest peaks at $\overline{\mu}x=3.5$ and $\overline{\mu}x=3.75$ are perfectly resolved, showing the high spatial resolution of the method, even in the presence of absorption. We also note that the contrast between the object $\delta\mu(x)$ and the background, $\delta\mu/\overline{\mu}$, was chosen less than 0.1%, stressing the good contrast of the method.

Detectable differences between the reconstructed image and the actual $\mu(x)$ arise mainly when the scattering properties of the object differ significantly from the background such that the object scatters the wave more than once and first-order perturbation theory is less accurate. This is shown in Fig. 1(b) where the object's scattering strength is comparable to its background, with $\delta\mu/\overline{\mu}$ near 1. Even in such a case, however, the present imaging scheme it still relatively reliable, and narrow structures can be detected.

Note that, while the preceding analysis used the reflected signal, a similar extension of this technique to transillumination geometries is desirable. Also, while we have presented the case of an object that differs from its environment by a small inhomogeneity in the scattering profile, a similar approach seems reasonable if the object differs instead by its absorptive properties.

The impact of the present work is easily shown if one considers the broad range of disciplines in which applications suggest themselves, such as in materials science or in fields with large length scales, such as meteorology or oceanography using intensity modulated sound waves. The most immediate impact of this technique is possible in biomedical imaging, where the intensity of the laser light is routinely modulated in time for clinical studies using photon density waves. Recent experimental work in this field has been carried out at a few selected modulation frequencies in cw approaches [9], or with time gating [10] in pulsed arrangements.

Several open question can now be addressed. The first and most obvious one is how this imaging is generalized to arbitrary two-and three-dimensional systems based solely on frequency scanning. While we have shown how such imaging is possible in quasi-one-dimensional situations, extension to more general cases will be addressed in future studies. In fact, ongoing studies for two-dimensional scattering geometries are nontrivial as the forward problem in the Boltzmann equation cannot be expressed in a closed form propagator solution and one might have to rely on the diffusion approximation, which is known to become unreliable for large modulation frequencies, or on an iterative approach [11]. The use of the proposed frequency scanning method in combination with a number of spatially separated sources and/or detectors is a promising area for future studies to improve the efficiency of the imaging.

A further area of interest involves adaptation of the current method to media in which the embedded object is characterized by periodic structures in its scattering or absorption profiles. Under these conditions, modulation frequency scanning could lead to resonance-like reflection or transmission signals. Recall the four general imaging categories mentioned in the introduction: transillumination, echo, resonance, and wave scattering. Thus the approach proposed in this paper suggests applications in three of the four broad categories and holds promise to become a useful tool in the probing of materials with waves.

One might also consider the possibility of multiple carrier frequencies, which introduces another new controllable degree of freedom to the imaging scheme. The contrast among frequency scans with separate colors will provide a more robust method with greater object discrimination or resolution. Another question concerns the noise sensitivity typical in most imaging techniques based on diffuse scattering, which could be of concern in the implementation of any imaging device based on this technique. We expect that these interesting questions will be addressed in future experimental and theoretical works.

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